Beyond the Conventional Quark Model: Using QCD Sum Rules to Explore the Spectrum of Exotic Hadrons

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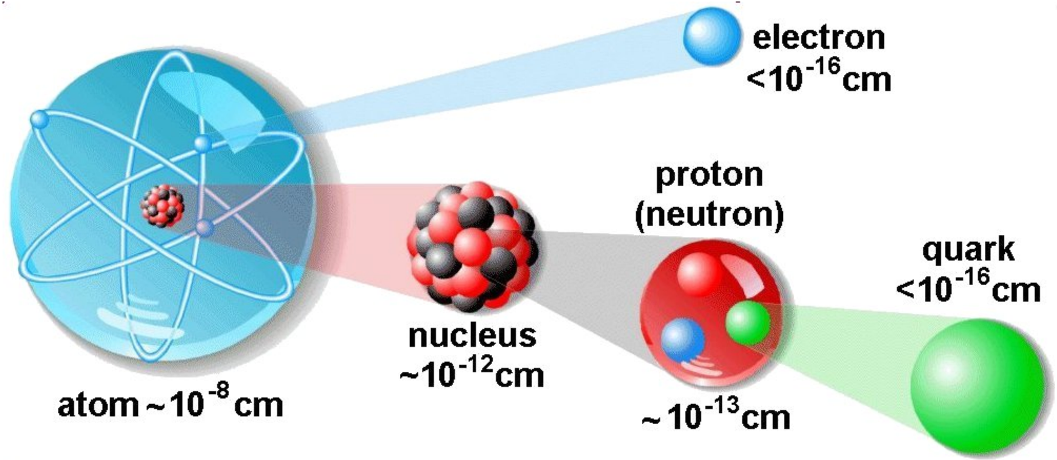
Outline

Structure of Matter



The Standard Model

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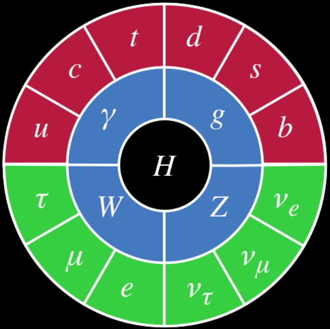
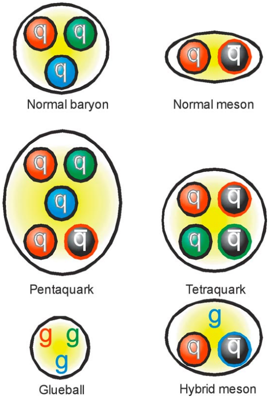


Image credit: *Particle Fever*, Dir. Mark Levinson and David Kaplan, Anthos Media, 2013. Film.

What is an exotic hadron?

- QCD - Quantum Chromodynamics
- Principle of colour confinement allows for the existence of any colourless bound states.
- Hybrid meson: meson with a “valence gluon”.



Why are we interested in exotics?

- XYZ Resonances
- GlueX (JLab)
- $Y(4260)$ $\bar{c}c$ hybrid candidate observed by BaBar (BABAR Collaboration, Phys. Rev. Lett. 95, 142001).
- Planned experiments: PANDA (FAIR).
- $Z_c(4430)$ four-quark state (Belle Collaboration, Phys. Rev. D 90, 112009).
- $P_c(4450)^+$ and $P_c(4380)^+$ five-quark states (LHCb Collaboration, Phys. Rev. Lett. 115, 072001).

QCD Sum-Rule Methodology

Connecting QCD Theory and Hadron Phenomenology

$$\Pi\left(Q^2\right) = \frac{1}{\pi}\int ds\, \frac{\rho^{\text{had}}\left(s\right)}{s+Q^2} + \dots$$

Quark-hadron Duality

Connecting QCD Theory and Hadron Phenomenology

$$\boxed{\Pi\left(Q^2\right)} = \frac{1}{\pi}\int ds\,\frac{\boxed{\rho^{\text{had}}(s)}}{s+Q^2} + \dots$$

QCD side

Quark-hadron Duality

Hadronic side

Quarks: Operator Product Expansion (OPE)

Correlators calculated within the Operator Product Expansion (OPE):

$$\lim_{x \to y} \mathcal{O}_1(x)\mathcal{O}_2(y) = \sum_n C_n(x-y)\mathcal{O}_n(x)$$

Quarks: Operator Product Expansion (OPE)

Correlators calculated within the Operator Product Expansion (OPE):

$$\lim_{x \to y} j_\mu(x) j_\nu(y) = C_1(x-y) + C_3(x-y)\langle m\bar{q}q\rangle + C_4(x-y)\langle G^2\rangle + \ldots$$

Quarks: Operator Product Expansion (OPE)

For our hybrid current $j_\mu(x) = g_s \bar{q}^a(x)\,\Gamma^\nu \mathcal{G}^n_{\mu\nu}(x)\,t^n_{ab} q^b(x)$,

$$
\begin{aligned}
\Pi_{\mu\nu}(q) &= i\int d^d x\, e^{iq\cdot x}\langle\Omega|\,T\,j_\mu(x)\,j^\dagger_\nu(0)\,|\Omega\rangle \\
&= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_v\left(q^2\right) + \frac{q_\mu q_\nu}{q^2}\Pi_s\left(q^2\right).
\end{aligned}
$$

Connecting QCD Theory and Hadron Phenomenology

$$\boxed{\Pi\left(Q^2\right)} = \frac{1}{\pi}\int ds\,\frac{\boxed{\rho^{\mathrm{had}}(s)}}{s+Q^2} + \dots$$

Quark-hadron Duality

QCD side

Hadronic side

Hadrons: Dispersion Relationship and Resonance Models

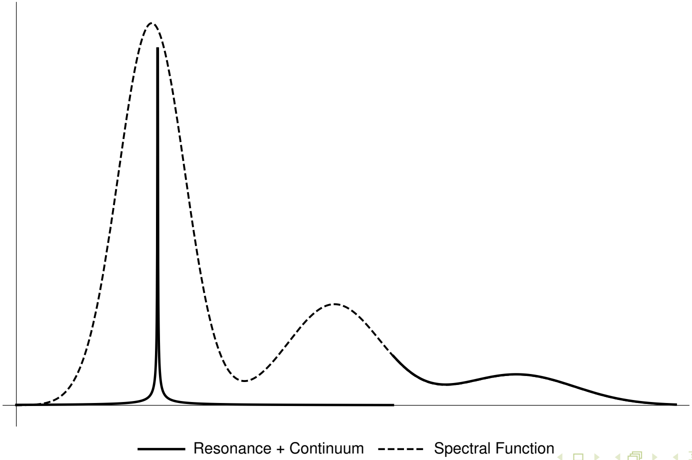
$$\Pi\left(Q^2\right) = \frac{1}{\pi}\int ds\, \frac{\rho^{\text{had}}(s)}{s+Q^2} + \dots$$

Hadrons: Dispersion Relationship and Resonance Models

Must model hadronic side to extract sum-rule

$$\rho^{\mathrm{QCD}}(t) = M_H^8 f_H^2 \delta(t - M_H^2) + \theta(t - s_0)\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(t)$$

Hadrons: Dispersion Relationship and Resonance Models



QCD Laplace Sum Rules

- M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **159** (1979)
- Dispersion relation

$$\Pi\left(Q^2\right) = \frac{1}{\pi}\int ds\, \frac{\rho^{\mathrm{had}(s)}}{s+Q^2} + (\mathrm{polynomials\,in\,Q^2})$$

relates information on the quarks on the left (our expansion of the correlation function) to hadronic features on the right.

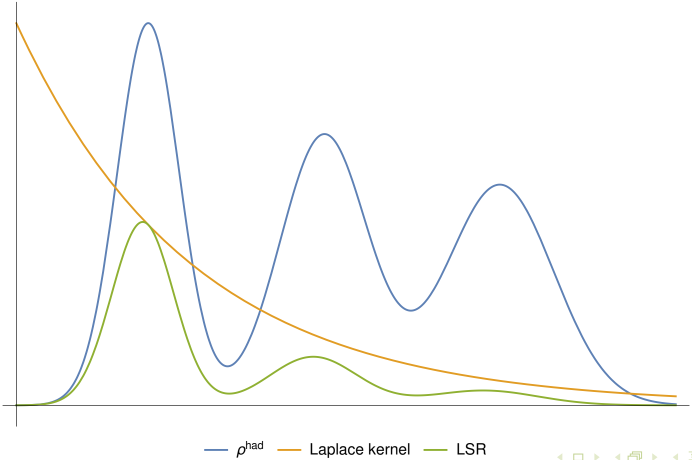
- To accentuate the ground state resonance and eliminate constant and polynomial terms, we apply the Borel transform $\hat{\mathcal{B}}$, given by

$$\hat{\mathcal{B}} = \lim \frac{1}{\Gamma(n)}\left(-Q^2\right)^n \left(\frac{d}{dQ^2}\right)^n, \; \left\{Q^2, n\right\} \to \infty, \; \frac{n}{Q^2} \equiv \tau.$$

Laplace Sum-Rules

- Borel transform may be expressed as an inverse Laplace transform
$$\frac{1}{\tau}\hat{\mathcal{B}}[f\left(Q^2\right)] = \mathcal{L}^{-1}[f\left(Q^2\right)]$$
- Forms the Laplace sum rule,
$$\mathcal{R}_k(\tau) \equiv \int_{M^2}^{\infty} dt\, t^k e^{-t\tau} \frac{1}{\pi}\rho^{\mathrm{had}}(t).$$
Using a resonance plus continuum model
$$\frac{1}{\pi}\rho^{\mathrm{had}}(t) = M_H^8 f_H^2 \delta(t - M_H^2) + \theta(t - s_0)\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(t)$$
we can extract the hadronic mass
$$M_H^2 = \frac{\mathcal{R}_{k+1}(\tau,s_0)}{\mathcal{R}_k(\tau,s_0)}.$$
where subtracted sum rule is
$$\mathcal{R}_k(\tau, s_0) = \mathcal{R}_k(\tau) - \int_{s_0}^{\infty} dt\, t^k e^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(t)$$

Laplace Sum-Rules



Borel Window

- LSR analyzed in a range of τ values where OPE converges, and analysis is τ-independent.
- τ upper bound:

$$\left|\frac{\mathcal{R}_k^{4D}(\tau,\infty)}{\mathcal{R}_k^{PT}(\tau,\infty)}\right| \leq \frac{1}{3} \qquad \left|\frac{\mathcal{R}_k^{6D}(\tau,\infty)}{\mathcal{R}_k^{4D}(\tau,\infty)}\right| \leq \frac{1}{3}$$

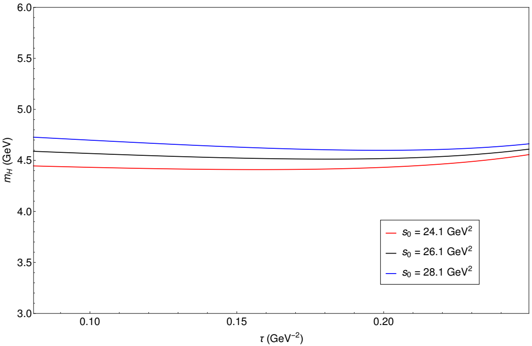
- τ lower bound:

$$\mathrm{PC}\left(s_0,\tau\right) = \frac{\int_{M_Q^2}^{s_0} e^{-t\tau}\,\mathrm{Im}\Pi(t)\mathrm{d}t}{\int_{M_Q^2}^{\infty} e^{-t\tau}\,\mathrm{Im}\Pi(t)\mathrm{d}t} \geq \frac{1}{10}$$

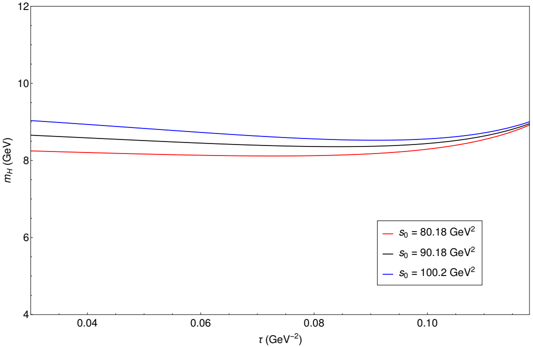
- Minimize

$$\sum\left(\frac{1}{m_H}\sqrt{\frac{\mathcal{R}_{k+1}(\tau_i,\, s_0)}{\mathcal{R}_k(\tau_i,\, s_0)}}-1\right)^2$$

Borel Window



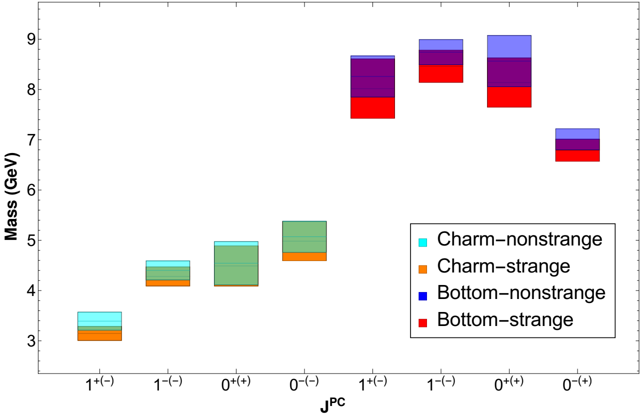
$0^{+(+)}$ Charm-light mass



$0^{+(+)}$ Bottom-light mass

Source: Ho, Harnett, and Steele, JHEP05(2017)149.

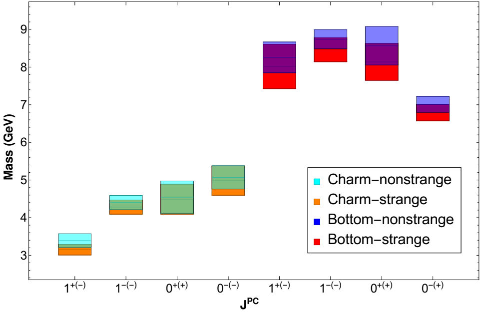
Results: Open-flavour Hybrid Mesons



Source: Ho, Harnett, and Steele, JHEP05(2017)149.

Results

- Predictions are heavier than previous GRW analysis, except in 1^+ charm-nonstrange and 0^- bottom-nonstrange channels.
- Similar spectrum hierarchy seen in charm and bottom channels.
- Discrepancies in 0^- consistent with predictions by Hilger, Krassnigg (Eur. Phys. J. A (2017) 53: 142).



Source: Ho, Harnett, and Steele, JHEP05(2017)149.

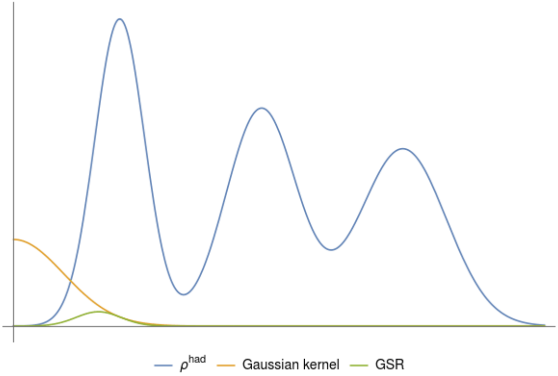
Gaussian Sum-Rules

Change of kernel $\rightarrow$ change of sum-rule
Originally, LSR: $\mathcal{R}_k(\tau) \equiv \int_{M^2}^{s_0} \mathrm{d}t\, t^k e^{-t\tau} \frac{1}{\pi} \rho^{\mathrm{had}}(t)$.

$$G_k(\hat{s},\, \tau,\, s_0) = \int_{t_0}^{s_0} \mathrm{d}t\, t^k \left(\frac{e^{-\frac{(\hat{s}-t)^2}{4\tau}}}{\sqrt{4\pi\tau}} \right) \frac{1}{\pi} \rho^{\mathrm{had}}(t)$$

What benefits does this have?

Gaussian Sum-Rules



Gaussian Sum-Rules

GSR can be imagined through the classical heat equation

$$\frac{\partial^2 G_k\left(\hat{s},\,\tau,\,s_0\right)}{\partial \hat{s}^2} = \frac{\partial G_k\left(\hat{s},\,\tau,\,s_0\right)}{\partial \tau},$$

reinterpreting the parameter $\hat{s}$ as “position”, the Gaussian width τ as “time”, and the GSRs $G_k(\hat{s},\,\tau,\,s_0)$ as “temperature”.

Results: Light Exotic Hybrid Mesons ($J^{PC} = 0^{+-}$)

Analysis of light exotic hybrid meson ($J^{PC} = 0^{+-}$). (arXiv:1806.02465 [hep-ph], submitted to PRD).

Models tested:

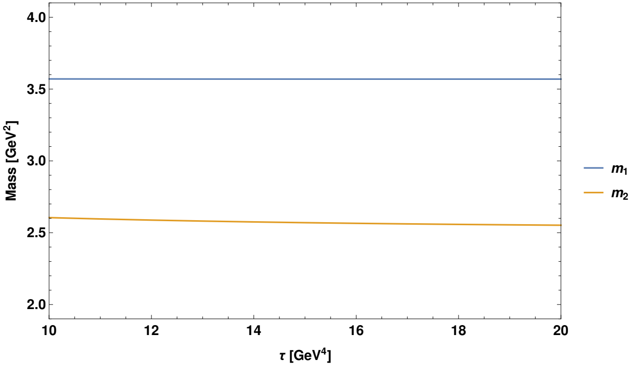
$$\text{Single-narrow Resonance} \rightarrow \frac{1}{\pi}\rho^{\text{had}}(t) = f^2\delta(t - m_H^2)$$

$$\text{Single-wide Resonance} \rightarrow \frac{1}{\pi}\rho^{\text{had}}(t) = \frac{f}{2m_H\Gamma}\left[\theta\left(t - m_H^2 + m_H\Gamma\right) - \theta\left(t - m_H^2 - m_H\Gamma\right)\right]$$

$$\text{Double-narrow Resonance} \rightarrow \frac{1}{\pi}\rho^{\text{had}}(t) = \left(f_1^2\delta\left(t - m_1^2\right) + f_2^2\delta\left(t - m_2^2\right)\right)$$

Results: Light Exotic Hybrid Mesons ($J^{PC} = 0^{+-}$)

Best results: double narrow resonance. Analysis gives $m_1 = 3.57 \pm 0.15\text{GeV}$ and $m_2 = 2.60 \pm 0.14\text{GeV}$.



Results: Light Exotic Hybrid Mesons ($J^{PC} = 0^{+-}$)

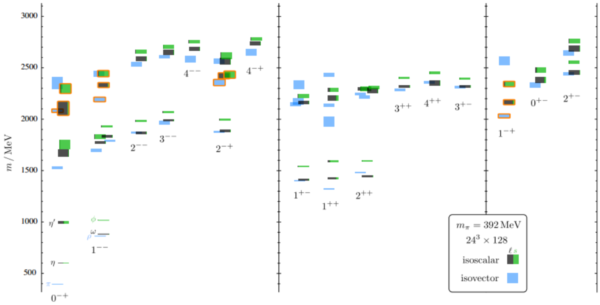


FIG. 11: Isoscalar (green/black) and isovector (blue) meson spectrum on the $m_\pi = 391$ MeV, $24^3 \times 128$ lattice. The vertical height of each box indicates the statistical uncertainty on the mass determination. States outlined in orange are the lowest-lying states having dominant overlap with operators featuring a chromomagnetic construction – their interpretation as the lightest hybrid meson supermultiplet will be discussed later.

Figure: Lattice results for spectrum of light mesons, including those with dominant gluonic character.
Source: J. Dudek *et.al.* , Phys. Rev. D 88, 094505 (2013)

Concluding Remarks

- Hybrid mesons are hadrons outside the traditional quark model, yet permissible within our current understanding of QCD.
- The QCD sum-rules framework provides a robust methodology to investigate properties of hadronic structures.
- LSR are well-suited for ground state analyses, while GSR allow for more complicated models.
- Our recent work focuses on hybrid mesons, and we have obtained predictions for open-flavour and light systems.
    - Open-flavour Hybrid Mesons $\rightarrow$ J. Ho, D. Harnett, T. G. Steele. JHEP05(2017)149
    - Light Exotic Hybrid Mesons $J^{PC} = 0^{+-}$ $\rightarrow$ J. Ho, R. Berg, Wei Chen, D. Harnett T. G. Steele. arXiv:1806.02465 [hep-ph]

Acknowledgements

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